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Optimizing vehicle routing and scheduling under time constraints

Оптимизација рутирања возила и редоследа са временским ограничењима

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Abstract: The Vehicle Routing Problem is essential in logistics for optimizing customer routes, especially in time-sensitive variants. This paper presents a two-stage algorithm for Vehicle Routing Problem with Time Windows. It effectively minimizes the number of vehicles, with transportation costs resulting just 0,38% above the best solution found on Solomon test instances. The approach limits search time to about 10 minutes, effectively balancing complexity and solution quality.
Keywords: vehicle routing, scheduling, time windows
JEL classification: C61

Сажетак: Проблем рутирања возила представља један од основних проблема у логистици рутирања потрошача, посебно када се укључи временска компонента проблема.У овом раду представљамо алгоритам са две фазе за оптимизацију проблема рутирања возила са временским ограничењима. Алгоритам на ефектан начин минимизује број возила, чији квалитет потврђујемо на класичним Соломоновим тест проблемима. Транспортни трошкови су већи тек за 0,38% од најбољих резултата пријављених у литератури. Уз лимитирање рада алгоритма 10 минута, на ефектан начин смо балансирали између комплексности проблема и квалитета решења.
Кључне речи: рутирање возила, утврђивање редоследа, временски прозори
ЈЕЛ класификација: C61

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Introduction

Vehicle Routing Problems (VRP) are the essence of every logistic model tackling transportation decisions. Assigning customers' requests to routes and seeking the satisfactory sequence has become an inevitable ingredient of contemporary decision-making tools. The variations of VRP are numerous, each originating from real-life applications. Following the lifestyle of modern customers, the most distinguished problems are time-related. Information about the approximate shipment arrival time increases consumer satisfaction, thus enhancing the quality of distribution.

The Vehicle Routing Problem with Time Windows (VRPTW) is a significant optimization challenge in logistics and transportation. It involves determining the optimal routes for a fleet of vehicles to service a set of customers within specified time windows. The complexity of VRPTW arises from its combinatorial nature and the need to balance multiple constraints, such as vehicle capacity, route length, and service times. Over the years, various solution approaches have been developed, each contributing to the advancement of the field.

The foundational work by Solomon in 1987 introduced VRPTW, providing benchmark problems and heuristic algorithms that have become standard references in the field. Solomon's algorithms laid the groundwork for subsequent research, offering initial solutions that addressed time window constraints and vehicle scheduling in a structured manner.

Furthermore, Solomon's work not only laid the foundation for future research but also stimulated the development of new methods and techniques for solving VRPTW. His benchmark problems continue to be widely used for testing new algorithms, enabling consistent evaluation of the performance of different approaches. Over the decades, the evolution of technology and the increase in data availability have enabled advanced analyses and the implementation of sophisticated models that better respond to the challenges of modern logistics systems.

Today, solving VRPTW is not just an academic endeavor but has direct applications in the industry. Optimizing routes while considering time windows can significantly reduce costs, improve efficiency, and increase customer satisfaction. For example, delivery services, food distributors, and pharmaceutical companies often use these models to ensure timely and reliable delivery of their products.

The aim of this paper is to provide an overview of current approaches to solving Solomon's problems in VRP, identify the advantages and disadvantages of different methods, and propose potential improvements for future work. Special attention will be given to methods combining heuristics and metaheuristics with exact methods to achieve a better balance between solution quality and computation time. This paper will also explore how advances in computing and algorithmic theory can further enhance the efficiency and applicability of VRPTW in real-world conditions.

1. Literature review

During the early 1990s, Desrochers et al. (1992) made significant contributions by developing efficient algorithms for solving large-scale VRPTW. Their work emphasized the use of Lagrangian relaxation techniques and branch-and-bound methods to improve computational efficiency.

In the early 2000s, the research expanded to include metaheuristic approaches, which provided more flexible and robust solutions to VRPTW. Bräysy and Gendreau (2005) conducted a comprehensive survey of metaheuristic algorithms for VRPTW, highlighting the effectiveness of tabu search, simulated annealing, and genetic algorithms in finding high-quality solutions for complex routing problems.

The Clarke-Wright savings algorithm, introduced by Clarke and Wright in 1964, remains one of the most widely used heuristics for solving VRPTW. This algorithm constructs initial solutions by iteratively merging routes based on cost savings, and it has been enhanced and adapted in numerous studies. For instance, Toth and Vigo (2002) reviewed various extensions and adaptations of the Clarke-Wright algorithm, demonstrating its continued relevance in modern VRPTW research. In the late 2000s and 2010s, hybrid algorithms became prominent, combining different heuristic and metaheuristic techniques to solve VRPTW more efficiently. Goel and Maini (2018) introduced a hybrid algorithm combining Ant Colony Optimization (ACO) and Firefly Algorithm (FA), leveraging the strengths of both algorithms to enhance solution quality and convergence speed.

Macrina et al. (2019) explored energy-efficient solutions for VRPTW by incorporating mixed vehicle fleets and partial battery recharging. Their research highlights the practical applications of green logistics, addressing both environmental sustainability and operational efficiency. This dual focus is crucial in today's logistics environment, where there is a growing emphasis on reducing carbon footprints. Goel et al. (2019) addressed the issue of stochastic customer demands and service times, presenting models and solutions that account for real-world uncertainties. Their work improves the robustness of vehicle routing solutions, making them more adaptable to dynamic environments where demand and service times can vary unpredictably. Jiang et al. (2020) proposed a hybrid multiobjective evolutionary algorithm based on variable neighborhood search for solving VRPTW involving hazardous materials (HazMat). Their focus on safety and efficiency provides a comprehensive solution for complex routing problems.

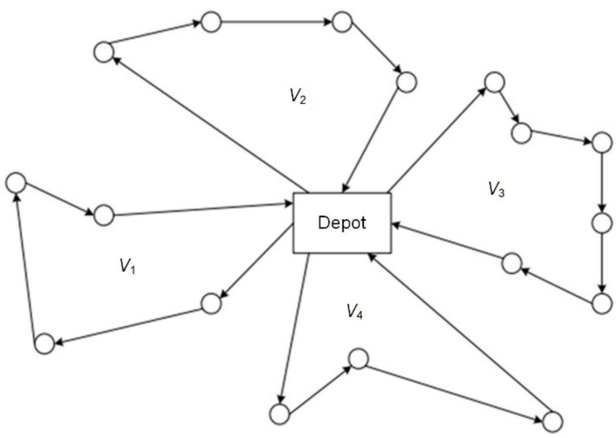
The integration of machine learning techniques into VRPTW is a more recent development. Julie Poullet (2020) explored the use of clustering and reinforcement learning to solve large-scale VRPTW. By leveraging these advanced computational methods, her research demonstrated significant improvements in efficiency and solution quality, paving the way for future studies to incorporate machine learning in logistics optimization.

These contributions underscore the evolution of VRPTW solutions from foundational heuristics to sophisticated hybrid and machine learning-based approaches. As the field continues to advance, future research is likely to focus on integrating real-time data, further enhancing the adaptability and robustness of VRPTW solutions in diverse and dynamic environments in the direction of a broader field – city logistics. City logistics is advancing towards the integration of multi-echelon distribution systems to enhance efficiency and sustainability in urban environments. This approach includes incorporating time constraints specifically to mitigate traffic congestion challenges (as explored by Rekabi, et al. in their study on pharmaceutical supply chain networks with perishable items). Additionally, it addresses the complexities of delivering perishable goods (as discussed by Bala et al., 2017) and managing biomass logistics (as highlighted in studies such as those by Cao et al. 2021.). These advancements aim to optimize the movement of goods within cities while considering operational limitations and environmental impacts, thus fostering smarter and more resilient urban logistics systems. For more details about literature on VRP and its variations, one can see Konstantakopoulos et al. (2022). Metaheuristic algorithms have been widely applied across various fields, not only transportation problems. Petrović et al. (2024) review the mathematical applications in economics. Andrijević et al. (2024) discuss the use of neural networks in energy consumption analysis, while Radak et al. (2024) illustrate the application of genetic algorithms for portfolio optimization.

2. Problem definition

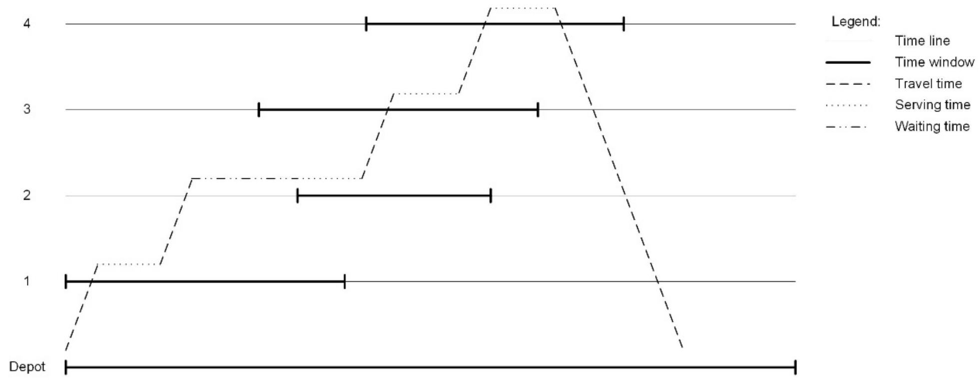
This section contains formal problem definition of VRPTW. Let D denote the depot and let $(a, b), a < b, a, b > 0$ be the time window of the depot. Denote with V a set of homogeneous vehicles of capacity C. With $\{1,2,\dots,n\}$ we will denote the set of customers, each of them with a time window $(a_i, b_i), a_i < b_i, a_i, b_i > 0$, quantity q_i, and a serving time $s_i, \forall i \in \{1,2,\dots,n\}$. Each route starts and ends at depot, with operating time between $[a, b]$. This means that total route time, including traveling, waiting and serving time, is at most $b - a$. Figure 1 depicts a depot, a set of 16 customers and four vehicles with accompanying routes.

Figure1: Vehicle routing scheme



Let t_i be the time of arrival of the vehicle at customer i. If it arrives before a_i, it will have to wait. Leaving time from customer i is $max\{t_i, a_i\} + s_i$. Total quantity of all customer requests on route must not exceed the vehicle capacity C. Number of all defined routes cannot be greater than the number of vehicles $|V|$. On Figure 2 we made an illustration of one possible route starting and ending at the depot and satisfying requests of four customers. We present time flow: the traveling, waiting, and serving time with lines of different shapes. Each object on this route is represented with a timeline and time window. In our example, waiting time occurs only at customer 2.

Figure2: Time flow on a route



3. The algorithm

In this section, we describe the algorithm for solving a VRPTW based on the metaheuristic Simulated Annealing (SA). It is a relatively simple, yet robust optimization technique for solving a range of optimization problems. SA is inspired by the cooling process in thermodynamics that imitates the process of metal cooling. Slower cooling transforms liquid metal into a crystal, which corresponds to the exploration of the solution space guided from feasibility to global optima. The papers of Kirkpatrick et al. (1983) and Černý (1985) are considered to be the introduction of the SA algorithm for optimization challenges. Both papers addressed a well-known problem of combinatorial optimization, the Traveling Salesman Problem (TSP). The use of SA is applied in many combinatorial optimization problems with single or multiple objectives, see Suman (2016).

Suppose that during the exploration of the solution space, the algorithm reaches some state s_1, and that state s_2 is a new candidate state. Given the evaluation function $E(\cdot)$, the algorithm moves to state s_2 with probability $exp(-(E(s_2) - E(s_1))/T)$. The temperature T decreases with running time. Consequently, the algorithm always moves to a cheaper solution and accepts the more expensive solution with a decreasing probability. The latter is particularly important for overcoming the local optima.

With the multicriteria nature of VRPTW, we propose a two-stage algorithm:

Stage 0. Forming the feasible solution.

Stage 1. Minimization of employed vehicles.

Stage 2. Minimization of total travel costs.

The solution space in all stages is explored following the SA principle. To reach a feasible solution, our evaluation function is guided with travel costs. Once the feasibility is achieved, we consider only states that satisfy the vehicle capacity and customer and depot time window constraints, throughout the algorithm execution. The evaluation function is composed of travel costs increased with a special ingredient in the first stage. Namely, after every k iterations, we randomly choose a route and expand the evaluation function with a logarithm of the number of customers on the route. The procedure is repeated several times, and when the stopping criteria are satisfied, we move to the second stage. The stopping criteria is defined via the specific temperature T^{crit}. Finally, in the second stage, given the number of vehicles, we explore the solution space with an evaluation function based only on total travel costs. Solutions with lower transportation costs, but a higher number of vehicles, are not considered.

Neighboring solutions are created using one of the four transformations:

T1. A customer is removed from the current route and inserted in a new position.

T2. Two customers interchange their positions.

T3. Let s and t be two different customers on the same route. Without loss of generality, suppose s precedes t, and denote s_1 and t_1 as the predecessors of s and t, respectively, and s_2 and t_2 as the followers of s and t, respectively. So, the current route has the following structure: $D, \dots, s_1, s, s_2, \dots, t_1, t, t_2, \dots, D$ with $s_2 \neq t_1$. The new route is obtained by combining segments: $D, \dots, s_1, s, t, t_1, \dots, s_2, t_2, \dots, D$, where sub-route $t_1, \dots, s_2$ has different orientation from initial setting.

T4. Let s and t be two customers on different routes, with followers s_2 and t_2 respectively. Route containing s has a structure $D, \dots, s, s_2, \dots, D$, and the one containing t is $D, \dots, t, t_2, \dots, D$. The new routes are obtained by combining segments: $D, \dots, s, t2, \dots, D$, and $D, \dots, t, s2, \dots, D$.

4. Results

We check the quality of the proposed approach on classical Solomon benchmark instances. Problem instances are defined in Solomon (1987) considering three dimensions. The first one is geographical distribution. The authors identify three characteristic situations for customer distribution: R – random uniform distribution, C – clustered, and RC – semi-clustered. The second dimension is the time horizon. Instances with narrow time windows for both customers and the depot, denoted with 1, imply a short scheduling horizon. On the other hand, instances with wider time windows, also for both customers and the depot, are denoted with 2, and allow a long scheduling horizon. Finally, the third dimension is problem size, expressed by the number of customers: 25, 50, and 100 customers.

In this paper, we consider the problem instances with 100 customers with both short and long scheduling horizons, and all geographical settings. All experiments were performed on the i5-4440@3.10GHz. In Table 1, we summarize our findings. The first column represents the problem group, with the number of the test in parenthesis. The second column shows the number of successful experiments. We run each problem 10 times, and characterize the experiment as successful if the algorithm reaches the minimum number of vehicles reported in the literature. The best solution for each instance is compared with the best-known solution (bks) by calculating $(price - bks)/bks$, where $price$ is the price of the best-found solution and bks is the best-known solution for a particular instance (see https://www.sintef.no/projectweb/top/vrptw/100-customers/). Next, we calculate the average over the entire group, considering only successful experiments. Similarly, the last column contains the average using the same metric. However, instead of using only the best-found solution, we consider all results for a particular instance, and report the average of all experiments for the group.

Table 1: Results of the algorithm for Solomon test instances

Group	Number of successful tests	Best found gap	Average gap
C1 (9)	90	0.0072%	0.0084%
C2 (8)	80	0.0033%	0.0048%
R1 (12)	117	0.0000%	0.7738%
R2 (11)	109	0.0000%	0.5517%
RC1 (8)	80	0.0000%	0.3008%
RC2 (8)	80	0.0034%	0.4225%
Total:	556	0.0000%	0.3771%

We set the cooling scheme to obtain a working time of the algorithm of approximately 10 minutes. However, the time needed to reach the best solution varied across the instances. On average, 392 seconds was the time when the algorithm discovered the best solution. The number of successful experiments was 556, or 99,29%. Four experiments failed, meaning our algorithm failed to find the minimal number of vehicles. Those were the instances “r104” from group R1 in 3 out of 10 experiments and “r207” from group R2 in 1 out of 10 experiments.

Conclusion and future work

In this paper, we present a two-stage algorithm for VRPTW. The search procedure of the solution space is organized with a well-known metaheuristic procedure called Simulated Annealing. The approach leads to more than 99% success in matching the minimal number of vehicles, while lagging by an average of 0.38% from the best results in the literature. We have defined a cooling scheme to match approximately 10 minutes of working time. Although the working time does not look impressive by itself, we believe it is satisfactory given the problem complexity and quality of solutions. Unfortunately, apart from comparisons with the best-known solutions obtained through different approaches, it is difficult to perform a head-to-head comparison with individual algorithms. Authors generally focus on the best solutions, making it very challenging to assess how individual algorithms perform in average cases.

The presented algorithm has no practical limitations in execution, unlike, for example, algorithms based on mathematical programming, which require many resources for large-dimensional problems. However, the tests performed on Gehring & Homberger benchmark instances with 200 customers showed that our approach does not provide successful tests in a number of instances. The algorithm loses efficiency, and additional heuristic improvements are necessary.

In future work, we will try to implement a few approaches that could lead to better performance of the algorithm. We believe that parallelization of the search procedure could lead to lower time consumption, a more thorough search of the solution space, and possibly overcoming the local minimum. Another direction could be the implementation of different objective functions throughout the working phases of the algorithm. Some ideas could include reducing the waiting time of customers, increasing/reducing the number of customers on a route, or a variation in quantifying the number of customers on a route using other than a logarithmic function.

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