UDC:

DOI: 10.5937/AnEkSub2300031T

Original scientific article

Анали Економског факултета у Суботици – The Annals of the Faculty of Economics in Subotica
Vol. XX, No. XX, pp. XX-XXX
Received: 10/08/2023
Accepted: 05/12/2023
Published online:22/12/2023

Productivity measurements, production function and Divisia index numbers

Мерење продуктивности, производна функција и Дивисиа индексни бројеви

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Abstract: The objective of this study is the theoretical and methodological analysis of the total productivity index, as a measure of economic efficiency, assuming the existence of various inputs and various outputs. The defined index is a measure of the overall change in productivity. The obtained chain two-factor indices are rationalized Divisia indices.

The study actualizes the issues of economic efficiency, measurements of productivity and total productivity, promotes relevant literature in this field, offering a simple methodology with numerous applications at the same time. The presented concept is based on index numbers of total productivity of all factors. Using this approach, net output is compared to a weighted average of all factor inputs. Therefore, a significant part of the study is particularly the input weighting methodology, since it enables the appropriate connection of inputs and outputs, combining their different units of measure at the same time.

The results obtained by applying the presented methodology in this paper provide the basis for numerous empirical analyses of the level and dynamics of changes in the economies' efficiency. Due to its simplicity, this approach has several advantages over other similar approaches.

Keywords: total productivity, index number, economic efficiency, labor productivity, capital productivity
JEL classification: C43 D24 E23 O47

Сажетак: Циљ ове студије је теоријска и методолошка анализа индекса укупне продуктивности, као мере економске ефикасности, уз претпоставку постојања више инпута и више аутпута. Дефинисани индекс је мера укупне промене продуктивности. Добијени ланчани двофакторски индекси су рационализовани Дивисиа индекси.

Студија актуелизује питања економске ефикасности, мерења продуктивности и укупне продуктивности, промовише релевантну литературу из ове области, а истовремено нуди једноставну методологију која има бројне примене. Презентовани концепт се заснива на индексним бројевима укупне продуктивности свих фактора. Такав приступ укључује посматрање нето аутпута у односу на пондерисани просек инпута свих фактора. Стога значајан део студије чини управо методологија пондерисања инпута, јер омогућава повезивање инпута и аутпута, али истовремено и комбиновање њихових различитих јединица мере.

Резултати добијени применом методологије представљене у овом раду представљају основу за бројне емпиријске анализе нивоа и динамике промена ефикасности привреде. Због своје једноставности овај приступ има низ предности у односу на друге сличне приступе.

Кључне речи: тотална продуктивност, индексни број, економска ефикасност, продуктивност рада,

Introduction

The main goal of this paper is a concept that offers a simple methodology, resulting in different index numbers, as a measure of the change in the level of productivity. In order to measure the change in productivity, it is necessary to simultaneously monitor the changes in the level of output, as well as the used inputs. Such changes are relatively simple to measure if one input and one output are assumed, and much more complex if conditions are investigated in which there are multiple inputs and multiple outputs at the same time. The concept of index numbers is, actually, one of the solutions to these complex research operations. Index numbers are real numbers that measure changes in a set of related variables. They can be used as an indicator of total factor productivity, as TFP index numbers, but also as partial productivity indicators - input and output index numbers. Also, index numbers can be used to measure changes in prices and quantities, but also in comparative analyses at the level of companies, industries, sectors, regions or economies.

The application of specific methods and criteria for measuring the level and growth of economic efficiency implies previously clearly defined theoretical assumptions. The production efficiency and its interaction with the phenomenon of development is in the center of attention when talking about the problems of modern socio-economic development in general, and economic development in particular (Baldwin & Gu, 2008). Namely, efficiency is the level of performance of used resources, and it is expressed as the ratio of production results and invested inputs. This is exactly what productivity is, so productivity is a general indicator of production efficiency. Besides, productivity is an important indicator for comparing the quality of the economy between related business entities in the country, and between different countries as well (Deaton, 2010).

Economic theory offers various analytical bases for empirical productivity research. However, there is no one, unique and generally accepted indicator that measures and expresses economic efficiency. Also, productivity growth has multiple effects on economic development, because more efficient production is the basis for the general improvement of the economic conditions of any development. Efficiency is most often expressed and measured by labor productivity (Balk, 2001). In order to see the quantitative dimension of economic growth and development, it is necessary to express labor productivity, capital productivity, but also other partial indicators of growth. The indicator of total productivity is of special importance among them, as a summary indicator of development, as well as the indicator of efficiency of transforming resources into products and services (Atkin, Khandelwal, Osman, 2019). It can be said that total factor productivity (TFP) is an extremely complex phenomenon, which refers to the long term and is connected with almost all elements that determine the dynamics of economic life. This is precisely why the ability to measure TFP is important. However, measuring TFP is not a goal, but a means or basis for better organizing and managing production at different levels of the economy.

Economic theory offers different methodological procedures, as different indicators of productivity. Each of these indicators expresses changes in the observed phenomenon in a certain way, but at the same time has certain advantages and disadvantages.

1. Total productivity factors as a measure of economic efficiency

Exploring the causes and consequences of changes in productivity and the connection between productivity and other economic categories is a complex matter because it requires an analysis of the entire economic mechanism with numerous interactions of different factors. The number of factors that affect productivity requires, from the analysis, prior classification of these factors into relatively homogeneous groups, which significantly depends on the level, the aspect and the purpose of the analysis itself (Diewert, 2004).

Sometimes, the complex phenomenon of total (global) factor productivity is approached very simplistically and even incorrectly. It is wrong to view productivity as an isolated phenomenon, because the level of productivity is determined by numerous and different factors, and at the same time productivity has multiple effects on numerous other economic categories. It is a complex interactive relationship. Analyses that take into account all the complexity of this phenomenon are mostly limited to one segment of the problem or set of relationships that are established between productivity and other categories at a certain level of observation (Copeland & Shapiro, 2016).

The term “productivity” is generally used to express the relationship between output and inputs used. This conceptual approach results in a number of development indicators and significant advantages over other alternative concepts. The methodology proposed in this paper includes weighting procedures, which ensure the combination of different output and input units, but also their appropriate connection. The index numbers of “productivity of all factors” are then based on observations of *net outputs* and *weighted averages of capital and labor inputs*. Then, the changes in productivity and interactions to which they are conditioned are viewed primarily in a long-term context.

Productivity is, therefore, a complex phenomenon essentially related to all those forces that determine the dynamics of economic life. As a result of numerous and diverse factors, changes in productivity significantly affect the movement of production costs, investments, accumulation, employment, prices, wages, and have a significant role in the development and changes in the structure of production. If productivity is defined as “labor and capital productivity” or “efficiency that transforms a country’s resources into products and services”, then the level of productivity is a *summary indicator of a country’s level of development* (Hill, 2004).

Low or declining productivity growth rate legitimately induces numerous economic disturbances that are multiplicatively carried over to all segments of the economic structure with a number of negative consequences. Economic efficiency is confirmed in faster or at least equal growth of costs and production results. Therefore, a low productivity growth rate directly affects the increase in total costs. Higher costs produce high prices of manufactured goods, and thus a decrease in sales volume, which further reduces the volume of production per employee and reduces overall work activity. This has a feedback effect on the further decline in productivity and income, which results in an increase in inflation and capital ratio, unemployment and a general decline in living standards.

The total productivity index is defined as the ratio of production growth and investment growth of all factors weighted by their relative importance. This is a summary indicator of actual production efficiency. The growth rate of total productivity is actually a weighted average of the growth rates of labor productivity and capital productivity, and indicates an increase in production that is realized without additional investment of factors. One of the dilemmas is whether to put the effect of production in relation only to human labor or to the total costs of production. Recognizing the fact that not only human (labor) but also material factors are engaged in the production process, certain dilemmas arise in terms of understanding productivity. Labor is only a part of the costs in the production process, so labor productivity is only one of the forms of economy and a measure of business success of economic entities. However, it is necessary to ensure the reproduction of all expenses from the realized income. If global (total or overall) productivity is expressed as: Y/(K+L), the methodological question arises whether K and L can be added. Numerous authors give a negative answer to this question (Diewert, 2015) and consider that total productivity is a theoretical concept that is quantitatively indeterminate. "Total productivity remains a concept without quantitative certainty and thus practical applicability…provides broad analytical possibilities in economic research" (Althin, 2001).

In addition to measuring total factor productivity (TFP) and methodological possibilities and limitations of TFP measurement, this paper also discusses Divisia or chain-link indexes. Namely, it starts from the point of view that the Divisia indices are the most appropriate and most suitable for studying the sources of economic growth. "The great advantage of the Divisia index is alleged to be its "accuracy", that is, its capacity to combine time series of prices and quantities to give a true reflection of the height of a utility or production function over time" (Usher, 1974). The results of numerous studies show that the acceptability and accuracy of the Divisia index implies very restrictive conditions for their application. This paper considers the properties of the Divisia, or chain-link, index, as they relate to the argument that this is the most appropriate index for use in studying the sources of economic growth. Usher D. (1974) indicates that "misplaced confidence in the Divisia index has led to errors of interpretation that might otherwise have been avoided, and has given rise to a distorted view of the process of economic growth".

2. Methodology for measuring total factor productivity

We start from the definition that the production function is an analytical summary description of the relationship between production costs and the quantity of finished products (Dean, Harper & Sherwood, 1996). The initial assumption of this model of estimating qualitative development factors is a multifactorial production function (Jorgenson & Griliches, 1971) characterized by a constant rate of return:

$$Y = f(K, L, T) \tag{1}$$

If p_Y, p_K and p_L are the prices of output, capital and labor, it is possible to define the share of capital and labor inputs in the output (w_K i w_L):

$$w_K = \frac{p_K \cdot K}{p_Y \cdot Y} \qquad w_L = \frac{p_L \cdot L}{p_Y \cdot Y} \tag{2}$$

The necessary equilibrium condition is obtained by equality between all values of participation, on one side, and the output elasticity and corresponding input, on the other side:

$$w_K = \frac{\partial lnY}{\partial lnK}(K, L, T) \qquad w_L = \frac{\partial lnY}{\partial lnL}(K, L, T) \tag{3}$$

Assuming a constant rate of return, the sum of elasticities and value shares is 1. The rate of technical progress (w_T) can be defined as the output growth over time, assuming a constant capital and labor input:

$$w_T = \frac{\partial lnY}{\partial T}(K, L, T) \tag{4}$$

At a constant rate of return, technical progress can be expressed as the output growth rate minus the weighted average growth rate of capital and labor inputs (Madžar, 1981). Weights represent the value of the corresponding input shares:

$$w_K \frac{dlnK}{dT} + w_L \frac{dlnL}{dT} + w_T \tag{5}$$

This expression of the rate of technical progress represents the quantitative index of technical progress – *Divisia index* (Hulten, 2008). In modern research, the Divisia indices represent a continuous series of numbers which, through the potential production function, are connected to the basic structure of the economy. However, it is sometimes possible for index numbers, even without a full appreciation of the economic structure, to show its basic characteristics even when only data on prices and quantities are used. The Divisia index is widely used in theoretical discussions of productivity analysis, and has important applications elsewhere. Older applications of the Divisia stressed its discrete-time axiomatic properties.

Total output (Y) can also be expressed as a function of aggregate input (M). The production function then takes the form:

$$Y = g[M(K, L), T] \tag{6}$$

The function *g* is linearly homogeneous for the aggregate inputs of capital and labor, while the technical progress of Hicks is neutral (Haughwout, 1998):

$$Y = A(t) \cdot M(K, L) \tag{7}$$

The rate of technical progress depends only on time:

$$w_T = \frac{dlnA}{dt} \tag{8}$$

The growth rate of aggregate input is the weighted average growth rate of capital and labor input:

$$\frac{dlnM}{dt} = w_K \frac{dlnK}{dT} + w_L \frac{dlnL}{dT} + w_T \tag{9}$$

which represents the *Divisia input index*.

At a constant rate of return, the necessary condition of producer's equilibrium is that the prices of output and input are consistent with the equality between the value of output and the sum of the value of input:

$$p_Y \cdot Y = p_K \cdot K + p_L \cdot L \tag{10}$$

the obtained equation allows the *output price* to be expressed as a function *p* of *the input price*:

$$p_Y = p(p_K, p_L, t) \tag{11}$$

which represents a function of prices, i.e. the price of each aggregate can be expressed as a function of the prices of its components.

The rate of technical progress can be defined as the negative output growth over time at constant input prices:

$$w_T = -\frac{\partial lnp_Y}{\partial t}(p_K, p_L, t) \tag{12}$$

or as a rate of weighted average input prices reduced by the growth rate of output prices (where weights are the corresponding value shares of input):

$$\frac{dlnp_Y}{dt} = w_K \frac{dlnp_K}{dt} + w_L \frac{dlnp_L}{dt} - w_t \tag{13}$$

This equation represents the *Divisia price index of technical progress* (Balk, 2005).

If the output is a function of aggregate input, the output price can be expressed as a function of aggregate input price (p_M):

$$p_Y = -\frac{p_M(p_K, p_L)}{A(t)} \tag{14}$$

then the growth rate of aggregate input price can be expressed as a weighted average growth rate of input prices (K and L):

$$\frac{dlnp_M}{dt} = w_K \frac{dlnp_K}{dt} + w_L \frac{dlnp_L}{dt} \tag{15}$$

which represents the *Divisia price index of input*.

The main characteristic of the *Divisia index* is that the aggregate index represents the product of its price and quantity. At the same time, aggregate indices are equal to the sum of aggregate components. Besides, Divisia indices have a reproductive property.

The proposed methodology for measuring productivity is based on the model of production and technical changes, which allows the analysis of the sources of output growth of certain economic areas (sectors). The complete model implies the production function of the sector (Caves, Laurits & Diewert, 1982), where the output is a function of capital, labor, reproductive consumption and time input:

$$Y_i = f_i\left(K_i, L_i, T\right) \qquad i=1, 2, 3,\ldots \tag{16}$$

Based on the defined production functions, it is possible to generate index numbers for sector outputs (Y_i), capital, labor and reproductive consumption inputs (K_i, L_i, X_i), corresponding prices and sector productivity.

The analysis of substitution between primary factors of production and reproductive consumption enables the combination of sectoral production functions and necessary equilibrium conditions (Trivić, Todić, 2022). Equilibrium conditions are defined by the equations between the value shares of each input in the sector and the elasticities of the output in relation to that input:

$$\begin{aligned} w_X^i &= \frac{\partial lnY_i}{\partial lnX_i}\,(X_i, K_i, L_i, T) \\ w_K^i &= \frac{\partial lnY_i}{\partial lnK_i}\,(X_i, K_i, L_i, T) \\ w_L^i &= \frac{\partial lnY_i}{\partial lnL_i}\,(X_i, K_i, L_i, T) \end{aligned} \tag{17}$$

At constant returns on capital (if we start from this assumption), the sum of elasticity and value shares of all three inputs is 1, for each sector:

$$w_X^i + w_K^i + w_L^i = 1 \tag{18}$$

The elasticity coefficients depend on the time and input variables of the production functions of the sector. The analysis of changes in substitution coefficients over time represents defined (known) rates of technical progress for each sector, defined as the growth rates of a sector's output at constant inputs of all factors. Rates of technical progress, as well as the elasticity of sectors outputs and inputs, depend on inputs and time (Lipsey & Carlaw, 2004). Then, the rate of technical progress (w_t^i) can be defined for each of the *n* sectors as:

$$w_t^i = \frac{\partial lnp_Y}{\partial t}\,(X_i, K_i, L_i, T) \tag{19}$$

Assuming constant returns on capital, the rate of technical progress can be defined as the growth rate of the corresponding sector output minus the weighted average growth rate of inputs (X, K and L) of that sector, and the weights are given by the corresponding shares of factors:

$$\frac{dlnY_i}{dt} = \frac{dlnY_i}{\partial lnX_i}\cdot\frac{dlnX_i}{dt} + \frac{\partial lnY_i}{\partial lnK_i}\cdot\frac{dlnK_i}{dt} + \frac{\partial lnY_i}{\partial lnL_i}\cdot\frac{dlnL_i}{dt} + \frac{\partial lnY_i}{\partial t} = w_X^i\frac{dlnX_i}{dt} + w_K^i\frac{dlnK_i}{dt} + w_L^i\frac{dlnL_i}{dt} + w_t^i \qquad i=1, 2, 3,\ldots \tag{20}$$

The expression (20) represents *the Divisia quantitative index of sectoral rates of technical progress*.

If the production function for each sector individually defines the output Y_i as a function of the aggregate of total input (M_i), then

$$Y_i = g_i[M_i, (X_i, K_i, L_i), t] \tag{21}$$

The total M_i is linearly homogeneous with respect to the inputs X_i, K_i, L_i of that sector. This implies that the technical progress of the Hicks sector is neutral, so it is:

$$Y_i = A_i(t) \cdot M_i(X_i, K_i, L_i) \tag{22}$$

and technical progress is:

$$w_t^i = \frac{dln A_i(t)}{dt} \tag{23}$$

The growth rate of sectoral input aggregates can be expressed as a weighted average growth rate of individual inputs:

$$\frac{dlnM_i}{dt} = w_X^i \frac{dlnX_i}{dt} + w_K^i \frac{dlnK_i}{dt} + w_L^i \frac{dlnL_i}{dt} \tag{24}$$

The model set up in this way does not require the existence of aggregate input of the sector to determine the index of technical progress of the sector, just as it does not require Hicks neutrality of technical progress of the sector. Defining the productivity index for each sector assumes that the sector output (Y_i) can be expressed as a translog function of sectors inputs. *The productivity index* is a translog index of sectoral technical progress (Gordon & Griliches, 1997):

$$w_t^i = [lnY_i(t) - lnY_i(t-1)] - w_X^i[lnX_i(t) - lnX_i(t-1)] - w_K^i[lnK_i(t) - lnK_i(t-1)] - w_L^i[lnL_i(t) - lnL_i(t-1)]$$
(25)

Weights represent the average shares of sectoral inputs in the value of sectoral output. The assumption is that the output value is equal to the sum of the input values (Dean, Harper & Sherwood, 1996). Value shares can be calculated from data on output (Y_i) and its price (p_Y^i), inputs (X_i, K_i, L_i) and their prices (p_X^i, p_K^i, p_L^i).

It is possible to compare the productivity indices defined in this way with the productivity indices based on value added (w_{Vt}) (Coelli, Prasada & Battese, 1998). These indices are generated based on the translog output index as:

$$w_{Vt} = w_V^i \cdot w_t^i \tag{26}$$

Value added is equal to the sum of the value of capital and labor inputs (Triplett, 2004). Weights w_V^i are given through the average share of value added in the sectoral output value.

If the scalar index m of outputs in time t is denoted by Y_i, the scalar index m of inputs by M_t, and their derivatives by time by Y'/Y and M'/M, then the *total factor productivity index* (TFP) is:

$$TFP = {Y'}/{Y} - {M'}/{M} \quad (27)$$

The logical question is, what conditions should apply so the total productivity factor can be measured in this way? The set of satisfactory conditions is that there is a consistent index of aggregate output and a consistent index of aggregate input (Uguccioni, 2016). If a hypothetical separation of the additive type is further assumed, the production function of the form can be specified:

$$g(Y_t) = f(M_t, t) = 0 \quad \text{or} \quad g(Y_t) = f(M_t, t) \quad (28)$$

Assuming that the function *g* and *f* is characterized by a constant rate of return to a given quantity, which is not necessary but it is useful for data movement and calculation, then the *total productivity factor* is a partial derivative:

$$\varepsilon_{Ft} = \frac{\partial lnf(M,t)}{\partial t} \quad (29)$$

If the output of a specific form of the production function *f* at moments *t* and *t-1* is represented by Y_t and Y_{t-1}, and index numbers, as specific measures of output, by I_t and I_{t-1}, such index numbers are assumed to be correct if:

$$\frac{Y_t}{Y_{t-1}} = \frac{I_t}{I_{t-1}} \quad (30)$$

The index thus defined is correct for a homogeneous production function of the translog form (Diewert, 2005), which is equally valid for the price index. This provides a theoretical basis for the use of these indices in productivity analyses, and also explains the name “translog index”. Other expressions of index numbers are correct for other forms of functions and can also be used (McLellan, 2004).

3. Application and characteristics of methods of measuring total productivity

Macroeconomic research of this type represents an ex-post analysis. The basis of the analysis is the production function, as an organizational principle of measuring the relationship of productivity with other quantities. Using a two-factor production function, T is sometimes called “technology”, and includes all the elements that affect the output, in addition to changes in the physical volume of inputs of material factors. As intangible factors accumulate through investment in education, research and development, this “intangible capital” is the primary source of efficiency (Bils & Klenow, 2001). The impression is that T will show minor changes if intangible inputs are included in the function together with material ones (Tomat, 2006). The Cobb-Douglas production function is defined for a given level of technology. If changes in technology need to be adjusted to logarithmic time series Y, K, L, it is necessary to keep in mind the change of scalar A, as well as exponent **b**. The magnitude of their changes can be estimated by different methods.

The expression $(1+r)^t$ can be used instead of A_t. At the same time, **r** measures the average annual scalar rates A resulting from technological changes (Feder, 2017). The value of the coefficient **r** is then equal to the slope of the trend line adjusted by the index numbers of productivity obtained as ***the output ratio to the weighted geometric average of the input***:

$$Y_t/(L_t^b \cdot K_t^{1-b}) \tag{31}$$

The main disadvantage of the described procedure is that it gives *average productivity growth rates* for the observed period as a whole. Further development of this methodology aims to enable greater flexibility in changing the relationship between variables, which includes the possibility of estimating *annual productivity growth rates*.

According to the second method, the annual growth rates A_t can be obtained as the difference between the rate of change in output and the rate of change in weighted inputs (Trivić, 2004). By differentiating the production function, the following is obtained:

$$\Delta A/_A = -\Delta Y/_Y \cdot [b(\Delta L/_L) + (1-b)\cdot(\Delta K/_K)] \tag{32}$$

which results in an *annual productivity growth rate*.

Time series of productivity can be provided by successively linking the annual rates of change in the residual with the base period, which has a value of 100. *The rate of total factor productivity* is then:

$$Y/_{[b \cdot L + (1-b)\cdot K]} \tag{33}$$

Thus expressed *total factor productivity* is the ratio of *output* and *weighted arithmetic average of input*. The weights are not derived from the statistical production function, but the income shares of the factors are estimated. In estimating the values obtained by this (index) approach, the assumption of the starting point of the Cobb-Douglas production function on linear homogeneity with unit elasticity of substitution between factors is avoided (Atkinson, Cornwell & Honerkamp, 2003).

This evaluation procedure is simpler than a number of others. It is often used in productivity analyses and is consistent with the weighting procedure used in measuring the real product (Diewert & Nakamura, 2003). Unlike a number of others, this weighting procedure does not require assumptions about the tendency to reduce the marginal productivity factors and about the neutrality of technical progress.

Conclusion

The primary goal of this theoretical and methodological analysis is the definition of the total productivity index, as a measure of economic efficiency, assuming the existence of multiple-inputs and multiple-outputs. The defined index is a measure of total productivity change, as a combination of technological change, technological efficiency change and scale efficiency change. The term productivity is generally accepted to denote the relationship between output and associated inputs used in the production process. The main goal of measuring total productivity is to estimate the impact of investments and other variables that improve knowledge and enhance the growth of production (output). That is, the goal is to estimate the contributions of individual factors to production efficiency. The total factor productivity index starts from the production function with constant returns and necessary equilibrium conditions.

The question arises: why is productivity growth considered so significant? Namely, productivity growth is closely related to general economic growth, with a number of other economic aggregates, but also with the growth of living standards. Why or how? An increase in production can occur with an increase in inputs or with an increase in productivity. It should be borne in mind that productivity growth is only the production growth that is greater than the input growth, which is the essence of the method of estimating productivity growth. Defined as the ratio of production and combined labor and capital costs, this index expresses changes in real product and real input costs.

The analysis of general efficiency on the basis of the total productivity index has a number of advantages compared to the measure of technical progress obtained on the basis of the production function. They can be reduced to the following:

1. non-parametric form of testing the correlation, eliminates the difficulties of statistical evaluation;

2. weighting system based on the estimates of factor shares in the functional distribution is simpler than the parameters obtained based on production function;

3. certain data corrections on weight values in aggregation, as well as on factor consumptions are possible.

The central part of this paper is the analysis of the methodological possibilities of defining the output, input and total productivity indices. To that end, on the basis of general theoretical interdependencies and mathematical axioms, the formulas of the corresponding chain indices were defined. A consistent result of such an approach implies that the possibility of measurement and quality adjustment of physical capital input and labor input is analyzed with special attention. The results obtained by applying the methodology presented in this paper are the basis for further empirical analysis of the level and dynamics of changes in the efficiency of the economy as a whole. This approach is easy to apply, and allows precise delineation of rates of change in productivity and its changes. The methodology developed in this article has a number of applications.

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